Edexcel Further Maths A-level

Further Pure 1

Formula Sheet

Provided in formula book	Not provided in formula book

Vectors

Scalar product	$\boldsymbol{a} \cdot \boldsymbol{b} = \lvert\boldsymbol{a}\rvert\lvert\boldsymbol{b}\rvert \cos\theta$
Vector product	$\boldsymbol{a} \times \boldsymbol{b} = \lvert\boldsymbol{a}\rvert\lvert\boldsymbol{b}\rvert \sin\theta\ \hat{n} = (a_2b_3 - a_3b_2)\boldsymbol{i} + (a_3b_1 - a_1b_3)\boldsymbol{j} + (a_1b_2 - a_2b_1)\boldsymbol{k} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

Scalar triple product	$\boldsymbol{a} \cdot (\boldsymbol{b} \times \boldsymbol{c}) = a_1(b_2c_3 - b_3c_2) + a_2(b_3c_1 - b_1c_3) + a_3(b_1c_2 - b_2c_1)$	$= \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

Area of a general triangle ABC	$\text{Area} = \frac{1}{2}\lvert\overrightarrow{AB} \times \overrightarrow{AC}\rvert$
Area of a general parallelogram $ABCD$	$\text{Area} = \lvert\overrightarrow{AB} \times \overrightarrow{AD}\rvert$

Volume of parallelepiped	$V = \lvert\boldsymbol{a} \cdot (\boldsymbol{b} \times \boldsymbol{c})\rvert$
Volume of tetrahedron	$V = \frac{1}{6}\lvert\boldsymbol{a} \cdot (\boldsymbol{b} \times \boldsymbol{c})\rvert$

Vector equation of a straight line passing through point $\boldsymbol{a}$ and parallel to vector $\boldsymbol{b}$	$\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b}$ $(\boldsymbol{r} - \boldsymbol{a}) \times \boldsymbol{b} = \boldsymbol{0}$

Direction cosines of the line parallel to the vector $\boldsymbol{a} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$	$l = \cos\alpha = \frac{x}{\lvert\boldsymbol{a}\rvert}, \mathrm{m} = \cos\beta = \frac{y}{\lvert\boldsymbol{a}\rvert}, n = \cos\gamma = \frac{z}{\lvert\boldsymbol{a}\rvert}$
	$l^2 + m^2 + n^2 = 1$

Equation of a plane	$\boldsymbol{r} \cdot \boldsymbol{n} = p$ $ax + by + cz = d$ where $\boldsymbol{n} = (a, b, c)$ is the normal vector to the plane

Shortest distance between two skew-lines $\boldsymbol{r_1} = \boldsymbol{a} + \lambda\boldsymbol{b}$ and $\boldsymbol{r}_2 = \boldsymbol{c} + \mu\boldsymbol{d}$	$\left\lvert\frac{(\boldsymbol{a} - \boldsymbol{c}) \cdot (\boldsymbol{b} \times \boldsymbol{d})}{\lvert\boldsymbol{b} \times \boldsymbol{d}\rvert}\right\rvert$

Conic Sections

Parametric equation of a curve	$x = p(t), y = q(t)$

Parabola

Cartesian equation of a parabola	$y^2 = 4ax$
Parametric equations of a parabola	$x = at^2, y = 2at, t \in \mathbb{R}$
Focus, S, of a parabola	$S = (a, 0)$
Directrix of a parabola	$x + a = 0$
Eccentricity of a parabola	$e = 1$
Vertex of a parabola	$(0,0)$
Equation of the tangent to the general parabola	$ty = x + at^2$
Equation of the normal to the general parabola	$y + tx = 2at + at^3$

Hyperbola

Cartesian equation of a rectangular hyperbola	$xy = c^2$
Parametric equation of a rectangular hyperbola	$x = ct, y = \frac{c}{t}, t \in \mathbb{R}, t \neq 0$
Equation of the tangent to the general rectangular hyperbola	$x + t^2y = 2ct$
Equation of the normal to the general rectangular hyperbola	$t^3x - ty = c(t^4 - 1)$

Cartesian equation of a hyperbola	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
Parametric equations of a hyperbola	$x = \pm a\cosh t\,, y = b\sinh t\,, t \in \mathbb{R}$ OR $x = a\sec\theta\,, y = b\tan\theta,$ $-\pi \leq \theta < \pi, \theta \neq \pm\frac{\pi}{2}$
Foci of a hyperbola	$(\pm ae, 0)$
Directrices of a hyperbola	$x = \pm\frac{a}{e}$
Eccentricity of a hyperbola	$e > 1, b^2 = a^2(e^2 - 1)$
Equations of the tangent to the general hyperbola	$ay\sinh t + ab = bx\cosh t$ OR $bx\sec\theta - ay\tan\theta = ab$
Equations of the normal to the general hyperbola	$ax\sinh t + by\cosh t = (a^2 + b^2)\sinh t\cosh t$

Ellipse

Cartesian equation of an ellipse	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
Parametric equation of an ellipse	$x = a\cos t\,, y = b\sin t\,, 0 \leq t < 2\pi$
Foci of an ellipse	$(\pm ae, 0)$
Directrices of an ellipse	$x = \pm\frac{a}{e}$
Eccentricity of an ellipse	$0 < e < 1$
Equation of the tangent to the general ellipse	$bx\cos t + ay\sin t = ab$
Equation of the normal to the general ellipse	$ax\sin t - by\cos t = (a^2 - b^2)\cos t\sin t$

The t-formulae

When $t = \tan\frac{\theta}{2}$:	$\sin\theta = \dfrac{2t}{1+t^2}$	$\cos\theta = \dfrac{1-t^2}{1+t^2}$	$\tan\theta = \dfrac{2t}{1-t^2}$

Taylor Series

$$f(x+a) = f(a) + f'(a)x + \frac{f''(a)}{2!}x^2 + \frac{f'''(a)}{3!}x^3 + \cdots + \frac{f^{(r)}(a)}{r!}x^r + \cdots$$

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots + \frac{f^{(r)}(a)}{r!}(x-a)^r + \cdots$$

Limits

Given $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} \mathrm{g}(x) = \mathrm{M}$:	$\lim_{x\to a}\big(f(x)+g(x)\big) = L + M$
	$\lim_{x\to a} cf(x) = cL$
	$\lim_{x\to a} f(x)g(x) = LM$
	For $M \neq 0$, $\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{L}{M}$

Series Solution of $\frac{dy}{dx} = f(x,y)$

$$y = y_0 + (x-x_0)\left.\frac{dy}{dx}\right|_{x_0} + \frac{(x-x_0)^2}{2!}\left.\frac{d^2y}{dx^2}\right|_{x_0} + \frac{(x-x_0)^3}{3!}\left.\frac{d^3y}{dx^3}\right|_{x_0} + \cdots$$

When $x_0 = 0, y = y_0 + x\left.\frac{dy}{dx}\right|_0 + \frac{x^2}{2!}\left.\frac{d^2y}{dx^2}\right|_0 + \frac{x^3}{3!}\left.\frac{d^3y}{dx^3}\right|_0$

Methods in Calculus

Leibniz's theorem for n^{th} derivatives	$\frac{d^n y}{dx^n} = \sum_{k=0}^{n} \binom{n}{k} \frac{d^k u}{dx^k} \frac{d^{n-k} y}{dx^{n-k}}$

L'Hospital's Rule

For functions $f(x)$ and $g(x)$ if either

$$\lim_{x \to a} f(x) = \lim_{x \to a} g(x) = 0,$$

or $\lim_{x \to a} f(x) = \pm\infty$ and $\lim_{x \to a} g(x) = \pm\infty$

then, if $\lim_{x \to a} \frac{f(x)}{g(x)}$ exists, $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$.

If $\lim_{x \to a} f(x)$ exists, then $\lim_{x \to a} e^{f(x)} = e^{\lim_{x \to a} f(x)}$

Weierstrass Substitution

$$t = \tan\frac{x}{2}, \Rightarrow dx = \frac{2}{1+t^2} dt$$

Function	Substitution
$\sin x$	$\frac{2t}{1+t^2}$
$\cos x$	$\frac{1-t^2}{1+t^2}$
$\tan x$	$\frac{2t}{1-t^2}$
$\sec x$	$\frac{1+t^2}{1-t^2}$
$\operatorname{cosec} x$	$\frac{1+t^2}{2t}$
$\cot x$	$\frac{1-t^2}{2t}$

Numerical Methods

Euler's method for approximating solutions for first-order differential equations	$\left(\frac{dy}{dx}\right)_0 \approx \frac{y_1 - y_0}{h}$ $y_{r+1} \approx y_r + h\left(\frac{dy}{dx}\right), r = 0,1,2, \ldots$
Euler's method for approximating solutions for second-order differential equations	$\left(\frac{d^2y}{dx^2}\right)_0 \approx \frac{y_1 - 2y_0 + y_{-1}}{h^2}$ $y_{r+1} \approx 2y_r - y_{r-1} + h^2\left(\frac{d^2y}{dx^2}\right), r = 0,1,2, \ldots$

Midpoint method for approximating solutions for first-order differential equations	$\left(\frac{dy}{dx}\right)_0 \approx \frac{y_1 - y_{-1}}{2h}$ $y_{r+1} \approx y_{r-1} + 2h\left(\frac{dy}{dx}\right) r = 0,1,2, \ldots$

Simpson's Rule for $2n$ Strips of Width h

$$\int_a^b f(x)dx \approx \frac{1}{3}h(y_0 + 4(y_1 + y_3 + \cdots + y_{2n-1}) + 2(y_2 + y_4 + \cdots + y_{2n-2}) + y_{2n})$$

$$\int_a^b f(x)dx \approx \frac{1}{3}h(\text{(endpoints)}+4\text{(odd values)}+2\text{(even values)})$$